Prof. Frithjof Lutscher, University of Ottawa, MAT 1332, Winter 2009
Assignment 3, due February 9, 8:30am in class

Student Name _____ Student Number _____

DGD 1 (FTX 227) DGD 2 (CBY B012) DGD 3 (TBT 070) DGD 4 (MCD 121)

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature _____

Numbers in square brackets denote marks. No part marks will be given.

[2] 1. Does the following improper integral converge or diverge? If it converges, give its value.

$$\int_0^\infty \frac{1}{(2+5x)^4} dx.$$

[2] 2. Does the following improper integral converge or diverge? If it converges, give its value.

$$\int_1^3 \frac{5}{\sqrt{x-1}} dx.$$

[3] 3. Use the comparison test to deduce whether the following integral converges ([2]), and if it does, find an upper bound on the value ([1]).

$$\int_1^\infty \frac{1}{\sqrt{x}+e^x} dx.$$

[4] 4. A population of bacteria is increasing at a rate of $\frac{1000}{(2+3t)^{1.5}}$ bacteria per hour, starting from a population of 1000.
(a) [1] Write pure-time differential equation to describe the above situation.
(b) [2] Could this sort of growth be maintained indefinitely?
(c) [1] Would the population ever reach 2000?

[4] 5. Use separation of variables to solve the following nonautonomous differential equation with initial condition $b(0) = 10^6$:

$$\frac{db}{dt} = e^{-t} b.$$

Continued on page 2

[6] 6. Torricelli's law states that the volume of water in a tank with a leak at the bottom decreases according to

$$\frac{dV}{dt} = -2\sqrt{V}, \qquad V(0) = 16.$$

(a) [3] Find the solution of the equation.
(b) [1] Find the time t^* at which the tank is empty.
(c) [2] Compare to the simple exponential decay process

$$\frac{dW}{dt} = -2W, \qquad W(0) = 16,$$

by calculating $W(t^*)$?

[8] 7. Consider the autonomous differential equation

$$\frac{dx}{dt} = f(x) = x^2 - 5x + 6, \qquad x(0) = 0.$$

(a) [2] Find the steady states x_1^*, x_2^*.
(b) [2] Use the stability test to find the stability of x_1^*, x_2^*.
(c) [1] Draw the phase line diagram.
(d) [3] Use separation of variables to solve the equation explicitly.